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A new simple finite element method for free vibration and buckling analysis of symmetrically laminated beams

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ABSTRACT: In this paper, a new 2-node element is proposed for free vibration and buckling analysis of symmetrically laminated beams. The element's formulation is based on first order shear deformation theory (FSDT). For this aim, the deflection and rotation field of the element is selected from third and second order functions, respectively. Moreover, the shear strain is assumed to be constant along with the element. By establishing the total strain energy in the element and stationary with respect to shear strain, the explicit form of the shape functions of deflection and rotation fields of the proposed element, are obtained. It should be mentioned, by decreasing the element's thickness, these shape functions are approach to the Euler-Bernoulli shape's functions and the shear locking problem does not occurred in the element. By utilizing the obtained shape functions, the explicit form of the stiffness matrix are calculated for the element. On the other hand, by using the governing equation of the free vibration and buckling of the beam, the explicit form of the translation and rotary mass matrices, and geometric stiffness matrix of the element are obtained. Finally, several numerical tests fulfill to assess the robustness of the developed element. For this purpose, free vibration and buckling analysis of symmetrically laminated beams with different boundary conditions and aspect ratios, are performed. The results of the numerical tests demonstrate high accuracy and efficiency of the proposed element for free vibration and buckling analysis of laminated beams.

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1. INTRODUCTION

Laminated beams due to their properties such as strength, hardness and lightness, are widely used in the construction of various engineering structures such as civil engineering, mechanics and aerospace. So far, many theories have been proposed for the structures analysis. In the classical beams theory (CBT), is not considered the effect of shear deformation of beam. For the analysis of thick beams that the effect of shear deformations is effective, two theories are developed; the first-order shear deformation theory (FSDT) and the high-shear deformation theory (HSDT).

Free vibration analysis is one of the important issues in the analysis of laminated beams. So far, various approaches have been proposed to solve the analytical and numerical problems of free vibration of these beams. Khdeir and Reddy presented the analytical solution for free vibration analysis of cross-ply laminated beams with virus theory [1]. Similar to the free vibration case, buckling analysis of these structures, is attractive for the researchers due to their wide application in the design of structures. Khdeir and Reddy developed the analytical solution of refined beam theories to study the buckling behavior of cross-ply rectangular beams with arbitrary boundary conditions [2]. In 2012, Vo and Thai performed vibration and buckling analysis of composite beams with arbitrary lay-ups using refined shear deformation theory [3]. Mantari and Canales in 2016, presented an analytical solution for the buckling and free vibration analysis of laminated beams by using a refined and generalized shear deformation theory. They used Rayleigh quotient, and the Ritz method is used to approximate the displacement field [4]. In this year, Kahya presented a multilayered beam element based on first-order shear deformation theory (FSDT) for buckling analysis of laminated composite beams [5]. In 2017, Nguyen et al. based on trigonometric series, proposed a new analytical solution based on a higher-order beam theory for static, buckling and vibration of laminated composite beams [6]. Moreover, they presented the solutions for static, buckling and vibration of laminated composite beams based on Ritz method [7].

In this study, is proposed a 2-node beam element for free vibration and buckling analysis of symmetric laminated beams, based on first order shear deformation theory. In the second section of the paper, the finite element formulation of the element is presented. .In order to formulate the element, cubic displacement polynomial and quadratic rotational

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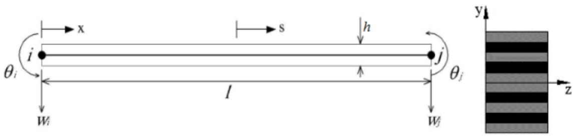

Fig. 1. Proposed laminated beam element

fields are selected. After the derivation of the finite element relations, the free vibration and the buckling analysis of the symmetric laminated beams has been evaluated with different boundary conditions. Numerical tests show the high accuracy of the proposed element in the free vibration and buckling analysis of these beams.

2. FINITE ELEMENT FORMULATION

In the finite element method, the deflection and rotation fields, are related to nodal displacements, by shape functions. "Fig. 1" shows the 2-node proposed laminated beam element. For calculating the shape functions, the deflection and rotation fields are selected from third and second order, respectively. Moreover, is assumed the shear strain to be constant. Therefore, these functions can be written as:

$$w=\frac{w_i}{2}(1-s)+\frac{w_j}{2}(1+s)+\beta_0 l\left(1-s^2\right)+\beta_1 ls\left(1-s^2\right) \tag{1}$$

$$\theta=\frac{\theta_i}{2}(1-s)+\frac{\theta_j}{2}(1+s)+\alpha_0\left(1-s^2\right) \tag{2}$$

$$\gamma=\gamma_0 \quad , \quad s=\frac{2x}{l}-1 \tag{3}$$

In these relations, β_1,β_0,α_0 and γ_0, are unknowns. In order to finding these parameters, the shear strain relation of the Timoshenko beam is used and By utilizing the shear strain value equal to γ_0, the subsequent equations will be available:

$$\gamma=\frac{dw}{dx}-\theta=\frac{2}{l}\cdot\frac{dw}{ds}-\theta \tag{4}$$

$$\gamma_0=\frac{2}{l}\left(-\frac{w_i}{2}+\frac{w_j}{2}-2\beta_0 ls+\beta_1 l-3\beta_1 ls^2\right)-\theta_i\left(\frac{1-s}{2}\right)-\theta_j\left(\frac{1+s}{2}\right)-\alpha_0\left(1-s^2\right) \tag{5}$$

In the present formula, the coefficients of the terms s and s^2 are equivalent to zero. Therefore, in the succeeding lines, β_1,α_0 are determined in terms of the unknown parameter γ_0:

$$\beta_0=\frac{1}{8}\left(\theta_i-\theta_j\right) \quad , \quad \beta_1=\frac{1}{6}\alpha_0 \tag{6}$$

$$\alpha_0=-\frac{3}{2}\left(\gamma_0-\frac{1}{l}\left(w_j-w_i\right)+\frac{1}{2}\left(\theta_i+\theta_j\right)\right) \tag{7}$$

It should be reminded that, in symmetric laminated beams, just the stiffness parameters D_{11} and A_{55} are nonzero. These parameter are calculated in the following way:

$$D_{11}=b\int_{-h/2}^{h/2} z^2\bar{Q}_{11}\,dz \tag{8}$$

$$A_{55}=bk_s\int_{-h/2}^{h/2}\bar{Q}_{55}\,dz \tag{9}$$

In these relations, b and k_s, are width of the beam and shear correction factor, respectively. As well as, $\bar{Q}_{11}^{(k)}$ and $\bar{Q}_{55}^{(k)}$ are determined as follows:

$$\bar{Q}_{11}^{(k)}=Q_{11}^{(k)}\cos^4\theta_k+2\left(Q_{12}^{(k)}+2Q_{66}^{(k)}\right)\times\sin^2\theta_k\cos^2\theta_k+Q_{22}^{(k)}\sin^4\theta_k \tag{10}$$

$$\bar{Q}_{55}^{(k)}=Q_{44}^{(k)}\sin^2\theta_k+Q_{55}^{(k)}\cos^2\theta_k \tag{11}$$

$$Q_{11}^{(k)}=\frac{E_1^{(k)}}{1-\nu_{12}^{(k)}\nu_{21}^{(k)}} \quad , \quad Q_{12}^{(k)}=\frac{E_2^{(k)}\nu_{12}^{(k)}}{1-\nu_{12}^{(k)}\nu_{21}^{(k)}} \tag{12}$$

$$Q_{22}^{(k)}=\frac{E_2^{(k)}}{1-\nu_{12}^{(k)}\nu_{21}^{(k)}} \quad , \quad Q_{44}^{(k)}=G_{23}^{(k)} \tag{13}$$

$$Q_{55}^{(k)}=G_{13}^{(k)} \quad , \quad Q_{66}^{(k)}=G_{12}^{(k)} \tag{14}$$

Where θ_k is the angle from global axis x to the principle material axis. Other required finite element relationships, can be found in [8].

3. NUMERICAL TESTS

In order to assess the accuracy and efficiency of the proposed element, some numerical problems have been analyzed and its results compared with results available in the literature. It should be mentioned, The 16 proposed element is used for analysis. The shear correction factors are assumed to be 5/6 and the following material properties are considered for each layer:

$$E_1=40E_2,\quad G_{12}=G_{13}=0.6E_2 \qquad G_{23}=0.5E_2,\quad \nu_{12}=0.25 \tag{15}$$

3.1. Free vibration analysis

In this section, the robustness of the proposed element is evaluated for free vibration analysis of laminated beams. For this aim, a three-layered symmetrically laminated beam with stacking sequence $\left[0^\circ,90^\circ,0^\circ\right]$ is analyzed for different thickness ratios and boundary conditions. The results of proposed elements, is compared with those of other researchers findings.

For convenience, the following non-dimensional natural

Table 1. Non-dimensional fundamental frequency of three layered beam with $\left[0^{\circ},90^{\circ},0^{\circ}\right]$

Method	$\frac{l}{h}$	Boundary condition			
		SS	**SC**	**CC**	**CF**
Nguyen et al. [7]	5	9.206	-	11.601	4.230
Murthy et al. [9]		9.207	10.238	11.602	4.230
Vo and Thai [3]		9.205	-	-	-
Khdeir & Reddy [1]		9.205	9.652	10.432	4.134
Proposed element		**9.216**	**9.664**	**10.447**	**4.135**
Nguyen et al. [7]	10	13.607	-	19.707	5.490
Murthy et al. [9]		13.611	16.600	19.719	5.491
Vo and Thai [3]		13.665	-	-	-
Khdeir & Reddy [1]		13.670	16.335	19.051	5.479
Proposed element		**13.679**	**16.350**	**19.075**	**5.480**
Nguyen et al. [7]	50	17.449	-	37.629	6.262
Vo and Thai [3]		17.456	-	-	-
Proposed element		**17.469**	**26.682**	**37.670**	**6.267**

Table 2. Non-dimensional buckling load of three layered beam with $\left[0^{\circ},90^{\circ},0^{\circ}\right]$

Method	$\frac{l}{h}$	Boundary condition			
		SS	**SC**	**CC**	**CF**
Kahya [5]	5	8.858	9.492	10.971	-
Mantari & Canales [4]		8.858	10.192	11.502	4.673
Nguyen et al. [6]		8.613	-	11.652	4.407
Khdeir & Reddy [2]		8.606	9.412	10.802	4.747
Proposed element		**8.612**	**9.418**	**10.813**	**4.748**
Kahya [5]	10	18.885	25.828	34.345	-
Mantari & Canales [4]		18.796	27.090	34.365	6.757
Nguyen et al. [6]		18.832	-	34.453	6.772
Khdeir & Reddy [2]		18.989	25.940	34.426	6.797
Proposed element		**19.004**	**25.974**	**34.513**	**6.780**
Nguyen et al. [6]	50	30.906	-	114.398	7.886
Proposed element		**30.931**	**61.288**	**114.830**	**7.887**

frequency will be given in term of the following form:

$$\bar{\omega} = \left(\frac{\omega l^2}{h}\right)\sqrt{\frac{\rho}{E_2}} \tag{16}$$

The results of proposed element along with the other published results, are presented in "Table 1". It is clearly seen that for all the aspect ratios and boundary conditions, the proposed element has rapid rate of convergence.

3.2. Buckling analysis

In order to assess the accuracy of the proposed element for buckling analysis of laminated beam structures, a three-layered symmetrically laminated beam with stacking sequence $\left[0^{\circ},90^{\circ},0^{\circ}\right]$ is analyzed for different thickness ratios and boundary conditions. For simplicity, the critical loads are given in non-dimensional form as follows:

$$\bar{P}_{cr} = \frac{P_{cr} l^2}{E_2 b h^3} \tag{17}$$

The results of the proposed element are compared with those of other researchers' findings, in "Table 2". The data listed in this Table confirm the element's high accuracy and rapid convergence.

4. CONCLUSION

In this study a simple 2-node element was proposed for free vibration and buckling analysis of symmetric laminated beams. The element was formulated based on FSDT theory. For this purpose, the displacement and rotation field of the element, has been selected from third and second order, respectively. In order to examine the accuracy and efficiency of the proposed element, several numerical tests were conducted. The results reveal that the suggested element has a high accuracy and convergence rate for free vibration and buckling analysis of symmetric beams.

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